



HOGERE ZEEVAARTSCHOOL ANTWERPEN

ONDERWIJSEENHEID 5

EXACTE WETENSCHAPPEN EN INFORMATICA

DRIEHOEKSMETING

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HOOFDSTUK 1

GONIOMETRIE EN DRIEHOEKSMETING

☞ 1.1. Bereken:

- | | |
|-----------------------------------|---|
| ▶1. $\sin 25^\circ 33,2'$ | ▶5. $\operatorname{tg} 25^\circ 33,2'$ |
| ▶2. $\cos 35^\circ 23,8'$ | 6. $\operatorname{cotg} 35^\circ 23,8'$ |
| 3. $\sin 125^\circ 31,5'$ | 7. $\operatorname{tg} 125^\circ 31,5'$ |
| 4. $\cos 56^\circ 16,4'$ | 8. $\operatorname{cotg} 56^\circ 16,4'$ |
| ▶9. $\arcsin 0,234$ | 13. $\arccos\left(-\frac{\sqrt{2}}{4}\right)$ |
| ▶10. $\arccos 0,234$ | 14. $\operatorname{arccotg} 1,678$ |
| ▶11. $\operatorname{arctg} 3,445$ | 15. $\operatorname{arctg} 0,342$ |
| 12. $\arcsin \frac{\sqrt{2}}{4}$ | 16. $\operatorname{arccotg} 0,112$ |

☞ 1.2. Bereken zonder rekenmachine:

- | | |
|-----------------------------------|-----------------------------------|
| 1. $\sin 75^\circ$ | 5. $\sin 15^\circ$ |
| 2. $\cos 75^\circ$ | 6. $\cos 15^\circ$ |
| 3. $\operatorname{tg} 75^\circ$ | 7. $\operatorname{tg} 15^\circ$ |
| 4. $\operatorname{cotg} 75^\circ$ | 8. $\operatorname{cotg} 15^\circ$ |

- ☞☞ **1.3.** De rechthoekszijden van een rechthoekige driehoek $\triangle ABC$ meten $a = 3\text{m}$ en $b = 4\text{m}$. Bepaal de lengte van de schuine zijde c . Bepaal verder de sinus, cosinus en tangens van de hoeken A en B . Bereken tenslotte de hoeken A en B .

Oplossingen - Solutions

1.1.

- | | |
|----------------------|-----------------------|
| 1. 0,43135 | 5. 0,47812 |
| 2. 0,81516 | 6. 1,40731 |
| 3. 0,81386 | 7. -1,40066 |
| 4. 0,55523 | 8. 0,66759 |
| 9. $13^\circ 32,0'$ | 13. $110^\circ 42,3'$ |
| 10. $76^\circ 28,0'$ | 14. $30^\circ 47,6'$ |
| 11. $73^\circ 48,8'$ | 15. $18^\circ 52,8'$ |
| 12. $20^\circ 42,3'$ | 16. $83^\circ 36,6'$ |

1.2.

- | | |
|--|--|
| 1. $\frac{\sqrt{6} + \sqrt{2}}{4} \sim 0,97$ | 5. $\frac{\sqrt{6} - \sqrt{2}}{4} \sim 0,26$ |
| 2. $\frac{\sqrt{6} - \sqrt{2}}{4} \sim 0,26$ | 6. $\frac{\sqrt{6} + \sqrt{2}}{4} \sim 0,97$ |
| 3. $2 + \sqrt{3} \sim 3,73$ | 7. $2 - \sqrt{3} \sim 0,27$ |
| 4. $2 - \sqrt{3} \sim 0,27$ | 8. $2 + \sqrt{3} \sim 3,73$ |

1.3.

$$c = 5\text{m},$$

$$\sin A = \cos B = \frac{3}{5},$$

$$\cos A = \sin B = \frac{4}{5},$$

$$\text{tg } A = \frac{3}{4},$$

$$\text{tg } B = \frac{4}{3},$$

$$A = 36^\circ 52,2',$$

$$B = 53^\circ 7,8'$$

HOOFDSTUK 2

BOLDRIEHOEKSMETING

© 2.1. (Labo T1) In de boldriehoek $\triangle ABC$ is

$$a = 110^{\circ}13,3', \quad b = 58^{\circ}21,2', \quad C = 90^{\circ}.$$

Bereken A , B en c .

⊙ **2.2.** (Labo T1) In de boldriehoek $\triangle ABC$ is

$$a = 105^{\circ}24,7', \quad B = 28^{\circ}36,2', \quad C = 90^{\circ}.$$

Bereken A , b en c .

⊙ **2.3.** (Labo T1) In de boldriehoek $\triangle KLM$ is

$$k = 78^{\circ}9,8', \quad L = 148^{\circ}18,4', \quad K = 90^{\circ}.$$

Bereken l , M en m .

⊙ **2.4.** (Labo T1) In de boldriehoek $\triangle ABC$ is

$$a = 51^{\circ}13,9', \quad c = 79^{\circ}51,6', \quad C = 90^{\circ}.$$

Bereken A , b en B .

⊙ **2.5.** (Labo T1) In de boldriehoek $\triangle KLM$ is

$$l = 141^{\circ}33,5', \quad L = 127^{\circ}9,7', \quad K = 90^{\circ}.$$

Bereken k , m en M .

⊙ **2.6.** (Labo T1) In de boldriehoek $\triangle ABC$ is

$$A = 71^{\circ}32,1', \quad b = 49^{\circ}23,7', \quad C = 90^{\circ}.$$

Bereken a , B en c .

⊙ **2.7.** (Labo T1) In de boldriehoek $\triangle PQZ$ is

$$P = 136^{\circ}24,9', \quad z = 63^{\circ}17,4', \quad Z = 90^{\circ}.$$

Bereken p , Q en q .

⊙ **2.8.** (Labo T1) In de boldriehoek $\triangle ABC$ is

$$A = 156^{\circ}17,4', \quad b = 72^{\circ}12,3', \quad B = 90^{\circ}.$$

Bereken a , C en c .

⊙ **2.9.** (Labo T1) In de boldriehoek $\triangle ABC$ is

$$b = 138^\circ 46,3', \quad B = 125^\circ 10,4', \quad C = 90^\circ.$$

Bereken a , C en c .

⊙ **2.10.** (Labo T1) In de boldriehoek $\triangle RST$ is

$$r = 35^\circ 34,3', \quad t = 45^\circ 48,2', \quad T = 90^\circ.$$

Bereken R , s en S .

⊗ **2.11.** In een boldriehoek zijn volgende afmetingen gegeven. Bereken de overige afmetingen.

$$\triangle ABC : A = 90^\circ, b = 155^\circ 12,6', c = 72^\circ 13,1'$$

$$\triangle ABC : A = 90^\circ, b = 55^\circ 12,6', B = 72^\circ 13,1'$$

$$\triangle ABC : A = 90^\circ, b = 125^\circ 15,3', B = 100^\circ 12,8'$$

$$\triangle ABC : A = 90^\circ, a = 134^\circ 56,9', b = 145^\circ 22,8'$$

$$\triangle ABC : A = 90^\circ, a = 21^\circ 56,9', b = 15^\circ 22,8'$$

$$\triangle ABC : A = 90^\circ, b = 35^\circ 34,3', C = 75^\circ 22,8'$$

$$\triangle ABC : A = 90^\circ, b = 135^\circ 34,3', C = 120^\circ 21,5'$$

$$\triangle ABC : A = 90^\circ, a = 95^\circ 34,3', C = 120^\circ 21,5'$$

$$\triangle ABC : A = 90^\circ, a = 85^\circ 24,3', C = 118^\circ 21,5'$$

$$\triangle ABC : A = 90^\circ, a = 86^\circ 24,3', C = 35^\circ 22,4'$$

$$\triangle ABC : A = 90^\circ, B = 85^\circ 24,3', C = 118^\circ 21,5'$$

☆ **2.12.** In een boldriehoek zijn volgende afmetingen gegeven. Bereken de overige afmetingen.

$$\triangle KLM : K = 90^\circ, L = 155^\circ 12,6', k = 72^\circ 13,1'$$

$$\triangle KLM : K = 90^\circ, L = 150^\circ 15,3', k = 70^\circ 14,6'$$

$$\triangle KLM : K = 90^\circ, k = 49^\circ 15,3', M = 103^\circ 24,6'$$

$$\triangle PQR : Q = 90^\circ, q = 45^\circ 24,6', R = 112^\circ 15,6'$$

$$\triangle KLM : K = 90^\circ, k = 79^\circ 56,7', L = 125^\circ 24,3'$$

$$\triangle KLM : K = 90^\circ, k = 79^\circ 56,7', L = 54^\circ 35,7'$$

$$\triangle ABC : A = 90^\circ, a = 73^\circ 15,3', b = 125^\circ 12,9'$$

$$\triangle KLM : K = 90^\circ, L = 147^\circ 15,3', k = 72^\circ 14,6'$$

$$\triangle ABC : A = 90^\circ, B = 67^\circ 38,8', C = 155^\circ 12,6'$$

© **2.13.** (Labo T2) In de boldriehoek $\triangle ABC$ is

$$a = 126^\circ 29,6', \quad b = 128^\circ 1,8', \quad c = 30^\circ 46,4'.$$

Bereken A , B en C .

© **2.14.** (Labo T2) In de boldriehoek $\triangle ABC$ is

$$A = 67^\circ 40,2', \quad b = 86^\circ 45,2', \quad c = 108^\circ 36,8'.$$

Bereken a , B en C .

© **2.15.** (Labo T2) In de boldriehoek $\triangle KLM$ is

$$k = 83^\circ 35,4', \quad l = 113^\circ 45,8', \quad m = 66^\circ 28'.$$

Bereken K , L en M .

© **2.16.** (Labo T2) In de boldriehoek $\triangle PQZ$ is

$$Q = 79^\circ 15,6', \quad p = 103^\circ 10,1', \quad z = 47^\circ 8,9'.$$

Bereken q , P en Z .

© **2.17.** (Labo T2) In de boldriehoek $\triangle RST$ is

$$T = 80^\circ 15,7', \quad r = 101^\circ 10,1', \quad s = 34^\circ 11,9'.$$

Bereken t , R en S .

© 2.18. (Labo T2) In de boldriehoek $\triangle PQZ$ is

$$p = 156^{\circ}42,2', \quad q = 33^{\circ}34,4', \quad z = 144^{\circ}6,6'.$$

Bereken P , Q en Z .

© 2.19. (Labo T2) In de boldriehoek $\triangle ABC$ is

$$B = 68^{\circ}35,4', \quad a = 58^{\circ}15,7', \quad c = 40^{\circ}15,8'.$$

Bereken b , A en C .

© 2.20. (Labo T2) In de boldriehoek $\triangle PQR$ is

$$p = 44^{\circ}32,1', \quad q = 104^{\circ}27,4', \quad r = 67^{\circ}8'.$$

Bereken P , Q en R .

© 2.21. (Labo T2) Bereken de kortste afstand tussen de twee plaatsen:

$$\begin{aligned} \text{Honolulu } 21^{\circ}19' \text{NB, } 157^{\circ}50' \text{WL,} \\ \text{Brussel } 50^{\circ}50' \text{NB, } 4^{\circ}21' \text{OL.} \end{aligned}$$

© 2.22. (Labo T2) Bereken de kortste afstand BI, IH, en HB.

$$\begin{aligned} \text{Bordeaux (B) } 44^{\circ}30' \text{NB, } 0^{\circ}20,4' \text{WL,} \\ \text{Istanbul (I) } 41^{\circ}1,2' \text{NB, } 28^{\circ}34,2' \text{OL,} \\ \text{Helsinki (H) } 60^{\circ}4,8' \text{NB, } 25^{\circ} \text{OL.} \end{aligned}$$

☆ **2.23.** In een boldriehoek zijn volgende afmetingen gegeven. Bereken de overige afmetingen.

$$\triangle PQZ : p = 86^\circ 45, 2', q = 108^\circ 36, 6', Z = 67^\circ 40, 2'$$

$$\triangle RST : r = 55^\circ 5, 5', s = 66^\circ 54, 3', t = 79^\circ 27, 3'$$

$$\triangle KLM : K = 64^\circ 26, 4', l = 76^\circ 24, 3', m = 109^\circ 1, 5'$$

$$\triangle RST : R = 59^\circ 24, 3', s = 71^\circ 26, 4', t = 111^\circ 1, 8'$$

$$\triangle PQZ : p = 86^\circ 45, 2', q = 108^\circ 36, 6', Z = 67^\circ 40, 2'$$

$$\triangle PRZ : Z = 65^\circ 31, 2', p = 126^\circ 2, 8', r = 28^\circ 36, 7'$$

$$\triangle RST : R = 65^\circ 31, 2', s = 126^\circ 2, 8', t = 25^\circ 1, 8'$$

$$\triangle KLM : K = 68^\circ 32, 2', l = 119^\circ 2, 8', m = 35^\circ 36, 7'$$

$$\triangle PRZ : Z = 64^\circ 32, 2', p = 121^\circ 2, 8', r = 30^\circ 36, 7'$$

$$\triangle KLM : K = 67^\circ 40, 2', p = 108^\circ 36, 8', r = 86^\circ 45, 2'$$

☆ **2.24.** In een boldriehoek zijn volgende afmetingen gegeven. Bereken de overige afmetingen.

$$\triangle RST : r = 56^\circ 22, 1', s = 65^\circ 54, 2', t = 78^\circ 27, 4'$$

$$\triangle RST : r = 53^\circ 5, 5', s = 103^\circ 4, 3', t = 59^\circ 1, 3'$$

$$\triangle KLM : l = 51^\circ 5, 6', m = 104^\circ 5, 3', k = 59^\circ 1, 3'$$

$$\triangle KLM : l = 53^\circ 5, 5', m = 103^\circ 4, 3', k = 59^\circ 1, 3'$$

$$\triangle RST : r = 55^\circ 5, 5', s = 66^\circ 54, 3', t = 79^\circ 27, 3'$$

$$\triangle RST : r = 56^\circ 22, 1', s = 65^\circ 54, 2', t = 78^\circ 27, 4'$$

$$\triangle RST : r = 51^\circ 5, 6', s = 104^\circ 5, 3', t = 59^\circ 1, 3'$$

Oplossingen - Solutions

2.1. $A = 107^{\circ}24,6'$, $B = 59^{\circ}57,5'$, $c = 100^{\circ}26,9'$

2.2. $A = 97^{\circ}18,6'$, $b = 27^{\circ}43,8'$, $c = 103^{\circ}36,3'$

2.3. $l = 149^{\circ}3,4'$, $m = 103^{\circ}50,2'$, $M = 97^{\circ}13,1'$

2.4. $A = 52^{\circ}22,6'$, $b = 73^{\circ}40,6'$, $B = 77^{\circ}8,2'$

2.5. $k = 51^{\circ}16,4'$, $m = 143^{\circ}0,6'$, $M = 129^{\circ}32,1'$
 $k = 128^{\circ}43,6'$, $m = 36^{\circ}59,4'$, $M = 50^{\circ}27,9'$

2.6. $a = 66^{\circ}15,6'$, $B = 51^{\circ}52,7'$, $c = 74^{\circ}48,6'$

2.7. $p = 141^{\circ}59,1'$, $q = 124^{\circ}47,1'$, $Q = 113^{\circ}9,7'$

- 2.8. $a = 157^{\circ}29,3'$, $c = 109^{\circ}19,1'$, $C = 97^{\circ}38,7'$
- 2.9. $a = 38^{\circ}8,3'$, $c = 126^{\circ}16,0'$, $A = 49^{\circ}59,4'$
 $a = 141^{\circ}51,7'$, $c = 53^{\circ}44,0'$, $A = 130^{\circ}0,6'$
- 2.10. $R = 54^{\circ}13,9'$, $S = 45^{\circ}56,4'$, $s = 31^{\circ}0,7'$
- 2.11.
- $B = 154^{\circ}7,5'$, $C = 82^{\circ}20,5'$, $a = 106^{\circ}5,8'$
 $C = 32^{\circ}21,6'$, $a = 59^{\circ}35,6'$, $c = 27^{\circ}29,5'$ $C = 147^{\circ}38,4'$, $a = 120^{\circ}24,4'$, $c = 152^{\circ}30,5'$
 $C = 162^{\circ}6,6'$, $a = 56^{\circ}4,3'$, $c = 165^{\circ}14,0'$ $C = 17^{\circ}53,4'$, $a = 123^{\circ}55,7'$, $c = 14^{\circ}46,0'$
 $B = 126^{\circ}36,5'$, $C = 46^{\circ}26,3'$, $c = 30^{\circ}51,3'$
 $B = 45^{\circ}12,0'$, $C = 46^{\circ}57,2'$, $c = 15^{\circ}51,1'$
 $B = 38^{\circ}5,3'$, $a = 70^{\circ}33,6'$, $c = 65^{\circ}50,9'$
 $B = 128^{\circ}2,4'$, $a = 62^{\circ}43,5'$, $c = 129^{\circ}55,2'$
 $B = 80^{\circ}35,3'$, $b = 79^{\circ}4,5'$, $c = 120^{\circ}49,0'$
 $B = 98^{\circ}26,5'$, $b = 99^{\circ}36,2'$, $c = 118^{\circ}41,9'$
 $B = 87^{\circ}27,1'$, $b = 85^{\circ}35,6'$, $c = 35^{\circ}17,6'$
 $a = 92^{\circ}29,2'$, $b = 84^{\circ}46,6'$, $c = 118^{\circ}27,5'$
- 2.12.
- $l = 156^{\circ}28,1'$, $M = 98^{\circ}1,7'$, $m = 109^{\circ}27,4'$
 $l = 152^{\circ}9,9'$, $M = 100^{\circ}56,0'$, $m = 112^{\circ}28,4'$
 $l = 164^{\circ}56,0'$, $L = 159^{\circ}56,0'$, $m = 132^{\circ}31,6'$
 $P = 149^{\circ}45,4'$, $p = 158^{\circ}58,8'$, $r = 138^{\circ}46,2'$
 $l = 126^{\circ}37,5'$, $M = 103^{\circ}48,0'$, $m = 107^{\circ}1,0'$
 $l = 53^{\circ}22,5'$, $M = 76^{\circ}12,0'$, $m = 72^{\circ}59,0'$
 $B = 121^{\circ}26,5'$, $C = 115^{\circ}13,9'$, $c = 119^{\circ}58,6'$
 $l = 148^{\circ}59,6'$, $M = 101^{\circ}5,8'$, $m = 110^{\circ}50,6'$
 $a = 152^{\circ}55,1'$, $b = 24^{\circ}54,0'$, $c = 168^{\circ}59,7'$

2.13. $A = 99^{\circ}20,3'$, $B = 104^{\circ}48,1'$, $C = 38^{\circ}54,1'$

2.14. $a = 70^{\circ}2,2'$, $B = 79^{\circ}17,1'$, $C = 111^{\circ}8,7'$

2.15. $K = 71^{\circ}2,8'$, $L = 119^{\circ}25,2'$, $M = 60^{\circ}45,6'$

2.16. $q = 91^{\circ}15,3'$, $P = 106^{\circ}53,2'$, $Z = 46^{\circ}5,5'$

2.17. $t = 93^{\circ}50,3'$, $R = 104^{\circ}16,8'$, $S = 33^{\circ}43,5'$

2.18. $P = 138^{\circ}40,8'$, $Q = 67^{\circ}24,2'$, $Z = 101^{\circ}50,7'$

2.19. $b = 52^{\circ}59'$, $A = 82^{\circ}34,9'$, $C = 48^{\circ}54,1'$

2.20. $P = 24^{\circ}49,1'$, $Q = 144^{\circ}34,9'$, $R = 33^{\circ}28'$

2.21. $106^{\circ}9,7'$

2.22.

$$BI = 21^{\circ}23,6', \quad IH = 19^{\circ}11,3', \quad HB = 21^{\circ}43'.$$

2.23.

$$\begin{aligned} P &= 79^{\circ}17,2', & Q &= 111^{\circ}8,5', & z &= 70^{\circ}2,2' \\ R &= 56^{\circ}23,8', & S &= 69^{\circ}6,4', & T &= 93^{\circ}9,1' \\ L &= 67^{\circ}44,4', & M &= 115^{\circ}49,4', & k &= 71^{\circ}20,8' \\ S &= 60^{\circ}2,8', & T &= 121^{\circ}27,0', & r &= 70^{\circ}21,5' \\ P &= 79^{\circ}17,2', & Q &= 111^{\circ}8,5', & z &= 70^{\circ}2,2' \\ P &= 128^{\circ}2,9', & R &= 27^{\circ}48,1', & z &= 110^{\circ}51,8' \\ S &= 126^{\circ}54,1', & T &= 24^{\circ}44,2', & r &= 113^{\circ}2,6' \\ L &= 123^{\circ}42,5', & M &= 33^{\circ}38,8', & k &= 102^{\circ}1,9' \\ P &= 126^{\circ}50,7', & R &= 28^{\circ}24,1', & z &= 104^{\circ}51,0' \\ P &= 111^{\circ}8,7', & R &= 79^{\circ}17,1', & k &= 70^{\circ}2,2' \end{aligned}$$

2.24.

$$\begin{aligned} R &= 58^{\circ}8,2', & S &= 68^{\circ}37,0', & T &= 91^{\circ}57,7' \\ R &= 30^{\circ}51,3', & S &= 141^{\circ}20,0', & T &= 33^{\circ}21,7' \\ K &= 27^{\circ}48,6', & L &= 25^{\circ}3,1', & M &= 148^{\circ}8,7' \\ K &= 33^{\circ}21,7', & L &= 30^{\circ}51,3', & M &= 141^{\circ}20,0' \\ R &= 56^{\circ}23,8', & S &= 69^{\circ}6,4', & T &= 93^{\circ}9,1' \\ R &= 58^{\circ}8,2', & S &= 68^{\circ}37,0', & T &= 91^{\circ}57,7' \\ R &= 25^{\circ}3,1', & S &= 148^{\circ}8,7', & T &= 27^{\circ}48,6' \end{aligned}$$