Chapter 2

Hydrodynamics

2.1 Flow and massconservation

By definition is flow the quantity of displaced mass per unit time

$$Q = \frac{dm}{dt}$$

This can be rewritten as ($m = \rho V = \rho Al$)

$$Q = \frac{dm}{dt} = \frac{d\rho Al}{dt} = \rho A \frac{dl}{dt} = \rho Av$$

Then if you keep in mind the law of massconservation, this will lead to Castelli's law

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

and if we consider that the density stays constant (fluid) we arrive at

$$v_1 A_1 = v_2 A_2$$

2.1.1 Viscosity

Viscosity is the measure for the way a fluid can flow. We can very well imagine that there is friction between the molecules of a certain fluid once they start to move. This friction leads to a tension. (A tension is by definition $\tau = \frac{F}{A}$.) This tension is a measure for the way a molecule can move so for the viscosity. By experiment we know that viscosity is due to speed so we can say that the viscosity is proportional to the gradient of the velocity and finally we get

$$\tau = \eta \frac{dv}{dx}$$

where η is the dynamic viscosity (unit Pa.s).

Mostly the kinematic viscosity

$$\nu = \frac{\eta}{\rho}$$

(unit $\frac{m^2}{s}$) is used.

In the fluidmechanics a lot of dimensionless numbers are used.

The Reynoldsnumber is one of them

$$Re = \frac{vl}{\nu}$$

where l is the characteristic length.

2.1.2 Flow and energyconservation

If we look at the law of conservation of energy we have learned in analytical mechanics we know

$$(E_k + E_p)_1 = (E_k + E_p + A)_2$$

We can rewrite this in terms of fluid mechanics. This means

$$E_k = \frac{mv^2}{2} = \frac{\rho V v^2}{2}$$

$$E_p = mgh = \rho V gh$$

The only energyform we still have to consider is the energy due to pressure pV and so we get at Bernouilli's equation

$$(\frac{\rho V v^2}{2} + \rho V gh + pV)_1 = (\frac{\rho V v^2}{2} + \rho V gh + pV)_2 + A$$

What do we have to fill out A which is the work done during the transformation of energy.This work has to be done due to the losses,like friction, induced in the proces. In our case these are the losses due to friction or the work that a pump has to supply during the proces. We can rewrite Bernouilli's equation in terms of height and then we become

$$(h + \frac{v^2}{2g} + \frac{p}{\rho g})_1 = (h + \frac{v^2}{2g} + \frac{p}{\rho g})_2 + \tau_{12}$$

whre τ_{12} are the losses during transformation of energy in the flow.

2.1.3 Losses during flow

If we want to calculate losses during a flow we have to use several possible formula's to make these calculations

Darcy:

$$h_w = \lambda \frac{lv^2}{d2g}$$

To calculate λ we use the formula of Colebrook

$$\lambda = \frac{1}{(-2log(\frac{2.51}{Re\sqrt{\lambda}} + \frac{\epsilon}{3.751d}))^2}$$

whre ϵ is a measure for the roughness of the wall.

Be carefull, we are calculating the flow but therefore we need the losses and for the losses we need to know the speed which is exactly what we are calculating. So we need to work iteratively, and start by making an educated guess.

2.1.4 Series- and parallel-flow

If we are working in **series** the flow stays constant and the losses are the total sum of the partial losses.

$$H_w = h_w$$

If we are working in **parallel** the flow will divide accordingly to the losses this means taht the flow will separate in the way that the losses over each parallel part will be the same

$$h_{w1} = ... = h_{wn}$$

and the flow will be

$$Q = \Sigma Q_n$$

2.1.5 Where does it all start?

Laws of conservation

mass	$\frac{\partial\rho}{\partial t} + div(\rho u) = 0$
x-momentum	$\frac{\partial\rho u}{\partial t} + div(\rho u\mathbf{u}) = -\frac{\partial p}{\partial x} + div(\mu gradu) + S_{Mx}$
y-momentum	$\frac{\partial\rho v}{\partial t} + div(\rho v\mathbf{u}) = -\frac{\partial p}{\partial y} + div(\mu gradv) + S_{My}$
z-momentum	$\frac{\partial\rho w}{\partial t} + div(\rho w\mathbf{u}) = -\frac{\partial p}{\partial z} + div(\mu gradw) + S_{Mz}$
energy	$\frac{\partial\rho i}{\partial t} + div(\rho i\mathbf{u}) = -pdiv\mathbf{u} + div(kgradT) + \Phi + S_i$

where S is the momentum source and Φ is the dissipation function. To be able to solve these equations we have to close the system and therefore we have to use equations of state.

Sometimes we can use the Navier Stokes equation for incompressible fluids

$$\rho(\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v}) = -\nabla p + \mu\nabla^2\mathbf{v} + f$$