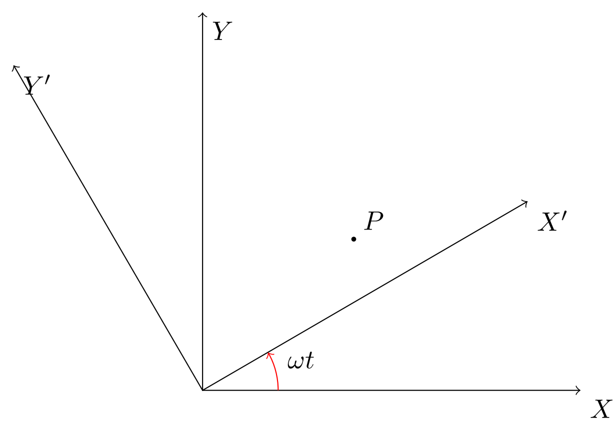
Massflow measurement by Coriolis force

1 Coriolis force

First we need to know how Coriolis force works. The Coriolis effect was named after Gustave Gaspard Coriolis who described this phenomenon for the first time in 1835. It is the explanation for the deflection of the trajectory of an object moving in a rotating frame.

A particle is moving on a straight line in a rotating reference frame.



The (X,Y) frame is the reference frame and the (X',Y') frame is the rotating frame.

At time t the rotating frame makes an angle ωt with the reference frame.

This means

$$P = x\vec{e_x} + y\vec{e_y} = x'\vec{e_{x'}} + y'\vec{e_{y'}}$$

The rotating frame coordinates expressed in the coordinates of the reference frame are

$$\vec{e_{x'}} = \vec{e_x} cos\omega t + \vec{e_y} sin\omega t$$

$$\vec{e_{y'}} = -\vec{e}_x sin\omega t + \vec{e_y} cos\omega t$$

Next step is to calculate the acceleration of our point P in the rotating frame. The acceleration is the second derivative (in time) of the position, being

$$\vec{a} = \frac{d^2\vec{p}}{dt^2}$$

The first derivative, thus the speed, in the rest frame

$$\frac{d\vec{p}}{dt} = \frac{dx\vec{e_x}}{dt} + \frac{dy\vec{e_y}}{dt}$$

Be carefull because we take the derivative of a product of a scalar with a vector. So we have two components which are in fact the change in module and the change in direction! For the x component this will be

$$\vec{v_x} = \frac{d(x\vec{e_x})}{dt} = \frac{dx}{dt}\vec{e_x} + x\frac{d\vec{e_x}}{dt}$$

This becomes

$$\frac{dx\vec{e_x}}{dt} = v_x\vec{e_x} + x\frac{d\vec{e_x}}{dt}$$

The first derivative of the change in direction of the speed vector becomes using the coordinates in the rotating frame

$$\frac{d\vec{e_{x'}}}{dt} = -\omega sin\omega t\vec{e_x} + \omega cos\omega t\vec{e_y} = \omega\vec{e_{y'}}$$

Accordingly we have for the y-component

$$\frac{d\vec{e_{y'}}}{dt} = -\omega cos\omega t\vec{e_x} - \omega sin\omega t\vec{e_y} = -\omega\vec{e_{x'}}$$

Now we find

$$\vec{v} = (v_{x'} - \omega y')e_{x'} + (v_{y'}\vec{\ } - \omega x')\vec{e_{y'}}$$

To calculate the acceleration we have to take the time derivative of the speed

For the x' component this becomes

$$(a_{x'} - \frac{d\omega}{dt}y' - \omega\frac{dy'}{dt})\vec{e_{x'}} + (v_{x'} - \omega y')\frac{d\vec{e_{x'}}}{dt} = (a_{x'} - \alpha y' - \omega v_{y'})\vec{e_{x'}} + (v_{x'} - \omega y')\omega\vec{e_{y'}}$$

We consider a constant rotational speed ω thus$\alpha = 0$ thus aariving at following equation for the acceleration,for the x' component

$$(a_{x'} - \omega v_{y'})\vec{e_{x'}} + (v_{x'} - \omega y')\omega\vec{e_{y'}}$$

For the y' component we arrive at following result

$$(a_{y'} + \omega v_{x'})\vec{e_{y'}} + (v_{y'} + \omega x')(-\omega\vec{e_{x'}})$$

Taking the sum of these vectors we arrive at

- first part: the linear acceleration

$$a_{x'}\vec{e_{x'}} + a_{y'}\vec{e_{y'}}$$

- second part: the centrifugal force:

$$-\omega^2(x'\vec{e_{x'}} + y'\vec{e_{y'}})$$

- third part: the coriolis acceleration

$$2\omega(-v_{y'}\vec{e_{x'}} + v_{x'}\vec{e_{y'}})$$

The last **vector** is the coriolis acceleration which can be written as

$$\vec{a_C} = -2\vec{\omega} \times \vec{v}$$

This is a vectorproduct so the result is a vector perpendicular to the surface in which $\vec{v}$ and $\vec{\omega}$ are oriented.

2 Intermezzo:urban legends and common myths

A widespread myth is the story of the swirling water above and below the equator. It is supposed that the rotational direction of water in an sink changes direction when crossing the equator. This is incorrect because other forces are influencing this direction and have far greater influence on the swirling than the coriolis force. One of this forces being the centrifugal force.For this we need to look at an dimensionless ratio called the Rossby number.

$$Ro = \frac{F_{CF}}{F_{cor}} = \frac{\frac{v^2}{R}}{2\omega v sin\phi}$$

with ϕ being the latitude

Let's take an example and consider $\phi = 78\deg$ latitude and a velocity of water of 1m/s and a sinkhole with a radius of 1m. ω being the rotational speed of the earth $(42000km/24h = 277m/s = 75 * 10^{-6}rad/s)$

We arrive at a very high Rossby number telling us that the coriolis force has almost no influence. We can calculate the radius of the sink hole if one wants to see the effect of the coriolis number. Let's calculate for Ro equal to 100 thus the centrifugal force has 100 times more influence then the coriolis force. Even then we need a sinkhole with a radius of 70m.

Where does the Coriolis force have an effect. Hurricanes and the trajectories of objects launched to overcome large distances. Airplanes also have to consider the effect in the calculations of their flightscheme.

3 Measurement

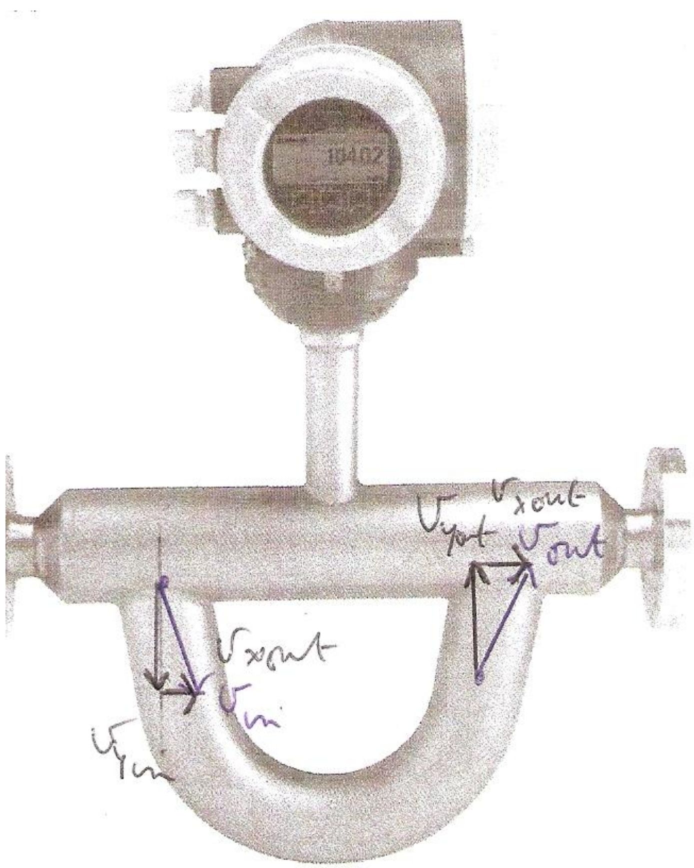


Figure 1: Coriolis mass flow meter(figuur uit procescontrol,nummer 6; 2016)

Most Coriolis mass flow meter are constructed with two parallel tubes attached with two flanges. They are brought into a vibrating mode, usually with a frequency between 800 and 1000 Hz. When there is no flow there is no resulting Coriolis force but once there is flow a resulting Coriolis force will occur. This force will create a difference in phase between the in- and outflow of the tubes. This phase difference is proportional to the massflow.

Try to find how it works by 'calculating' the vectors (and certainly their directions) drawn on the picture above.

3.1 Characteristics of the coriolismeasurement

In every coriolis mass flowmeasure you can find a temperature measurement because tubecharacteristics like diameter and length as well as flexibility depend on the temperature. The tube takes the temperature of the fluid or gas that runs through it thus taking the temperature of this fluid (gas) running through the tube. Also a density measurement is provided in the coriolis massflowmeter. We have measured three different quantities, temperature, massflow and density. With these variables we can calculate by means of software concentrations of specific elements in the flow. Because the frequency of a vibration depends on the mass, the density is calculated from the frequency of the vibration. The relation between frequency and mass is $m = \frac{1}{f^2}$

With a tolerance of about 0.05% this type of massflow measurement is the most precise on the market.

When this instrument is calibrated one has to take care that there are no airbubbles in the device. Avoiding the encapsulation of airbubbles is a matter of correct installing the device. Another danger of bubbles in the device is cavitation caused at high velocity and low pressure of the medium running through the coriolis massflowmeasurement.

Another problem is that the measurement tubes can foul by deposition or corrosion and that the density measurement is no longer correct although the flow can still be measured quite correct.